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The behavior of two parallel symmetry permeable interface cracks in a piezoelectric layer bonded to two half piezoelectric materials planes

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Abstract

In this paper, the behavior of two parallel symmetry permeable interface cracks in a piezoelectric layer bonded to two half piezoelectric materials planes subjected to an anti-plane shear loading is investigated by using Schmidt method. By using the Fourier transform, the problem can be solved with the help of two pairs of dual integral equations. These equations are solved using the Schmidt method. This process is quite different from that adopted previously. The normalized stress and electrical displacement intensity factors are determined for different geometric and property parameters for permeable crack surface conditions. Numerical examples are provided to show the effect of the geometry of the interacting cracks, the thickness and the materials constants of the piezoelectric layer upon the stress and the electric displacement intensity factor of the cracks. Contrary to the impermeable crack surface condition solution, it is found that the electric displacement intensity factors for the permeable crack surface conditions are much smaller than the results for the impermeable crack surface conditions.

Keywords: Piezoelectric materials layer; Schmidt method; Dual integral equations; Parallel interfacial crack

1. Introduction

It is well known that piezoelectric materials produce an electric field when deformed and undergo deformation when subjected to an electric field. The coupling nature of piezoelectric materials has attracted wide applications in electric-mechanical and electric devices, such as electric-mechanical actuators, sensors and structures. When subjected to mechanical and electrical loads in service, these piezoelectric materials can fail prematurely due to defects, e.g. cracks, holes, etc. arising during their manufacture process. Therefore, it is of great importance to study the electro-elastic interaction and fracture behavior of piezoelectric materials. Moreover, it is known that the failure of solids results from the cracks, and in most

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cases, the unstable growth of the crack is brought about by the external loads. So, the study of the fracture mechanics of piezoelectric materials is much important in recent research, especially when multiple interface cracks are involved.

In the theoretical studies of crack problems, several different electric boundary conditions at the crack surfaces have been proposed by numerous researchers. For example, for the sake of analytical simplification, the assumption that the crack surfaces are impermeable to electric fields was adopted by Deeg (1980), Pak (1990, 1992), Sosa and Pak (1990), Sosa (1991, 1992), Suo et al. (1992), and Gao et al. (1997), etc. In this model, the assumption of the impermeable cracks refers to the fact that the crack surfaces are free of surface charge and thus the electric displacement vanish insides the crack. In fact, cracks in piezoelectric materials consist of vacuum, air or some other gas. This requires that the electric fields can propagate through the crack, so the electric displacement component perpendicular to the crack surfaces should be continuous across the crack surfaces. Along this line, Zhank and Hack (1992) analyzed crack problems in piezoelectric materials. In addition, usually the conducting cracks which are filled with conducting gas or liquid are also applied to be a kind of simplified cracks models in piezoelectric materials by many researchers, such as McMeeking (1989) and Suo (1993). Dunn (1994), Zhang and Tong (1996), and Sosa and Khutoryansky (1999) avoided the common assumption of electric impermeability and utilized more accurate electric boundary conditions at the rim of an elliptical flaw to deal with anti-plane problems in piezoelectricity. They analyzed the effects of electric boundary conditions at the crack surfaces on the fracture mechanics of piezoelectric materials.

Layered materials can be used to manufacture high performance structures in order to achieve a high strength-to-weight ratio. Therefore, the analysis of laminated piezoelectric composite structures has attracted the attention of many researchers in recent years, such as Shen et al. (1999a,b) and Shen et al. (2000). Kim and Jones (1996) have studied the behavior of brittle fracture at the interface between two dissimilar piezoelectric materials. Beom and Atluri (1996) derived the complete form of stress and electric displacement fields of an interfacial crack between two dissimilar anisotropic piezoelectric media. The plane problem of a crack terminating at the interface of a bimaterial piezoelectric was treated by Qin and Yu (1997). In particular, control of laminated structures including piezoelectric devices was the subject of research by Tauchert (1996), Lee and Jiang (1996), Batra and Liang (1997), and Heyliger (1997). Many piezoelectric devices comprise both piezoelectric and structural layers, and an understanding of the fracture process of piezoelectric structural systems is of great importance in order to ensure the structural integrity of piezoelectric devices (Shindo et al., 1998; Narita et al., 1999; Chen et al., 1998). Recently, Soh et al. (2000) have investigated the behavior of a bi-piezoelectric ceramic layer with an interfacial crack by using the dislocation density function and the singular integral equation method. To our knowledge, the electro-elastic behavior of a piezoelectric ceramic with two parallel interface cracks subjected to an anti-plane shear loading has not been studied despite the fact that many piezoelectric devices are constructed in a laminated form by using the Schmidt method.

In the present paper, we consider the electro-elastic behavior of two parallel symmetry permeable interface cracks in a piezoelectric layer bonded to two same half piezoelectric materials planes subjected to an anti-plane shear is investigated using the Schmidt method (Morse and Feshbach, 1958). It is a simple and convenient method for solving this problem. Fourier transform is applied and a mixed boundary value problem is reduced to two pairs of dual integral equations. In solving the dual integral equations, the gaps of two crack surface displacement are expanded in a series of Jacobi polynomials. This process is quite different from that adopted in previous works (Han and Wang, 1999; Deeg, 1980; Pak, 1992; Sosa, 1992; Suo et al., 1992; Park and Sun, 1995; Zhang and Tong, 1996; Gao et al., 1997; Wang, 1992; Narita et al., 1999; Chen et al., 1998; Shen et al., 1999a,b; Shen et al., 2000; Kim and Jones, 1996; Beom and Atluri, 1996; Qin and Yu, 1997; Soh et al., 2000). The form of solution is easy to understand. Numerical solutions are obtained for the stress and electric displacement intensity factors for permeable crack surface conditions. Note that the conducting crack condition is a special case of the permeable crack considered by other researchers (Parton, 1976; Zhank and Hack, 1992). Another main objective of the present study is to investigate the effect of the layer thickness, the distance between two cracks and the material constants of the two dissimilar materials on the fracture behavior.

2. Formulation of the problem

Fig. 1 shows a layered structure made by bonding together two same half piezoelectric materials plane. The piezoelectric materials layers are layer 2 and layer 3 of thickness h, with two parallel interface cracks of length $2l$ created. A Cartesian coordinate system (x, y, z) is positioned with its origin at the center between two parallel interface cracks for reference purposes. Note that the z-axis is oriented in the poling direction of the piezoelectric materials, and the x–y plane is the transversely isotropic plane. Also note that all quantities with superscript k ($k = 1, 2, 3, 4$) refer to the upper half plane 1, the layer 2, the layer 3 and the lower half plane 4 as in Fig. 1, respectively. The constitutive equations for the mode III crack can be expressed as

$$\sigma_{jz}^{(k)} = c_{44}^{(k)} w_j'^{(k)} + e_{15}^{(k)} \phi_j'^{(k)} \tag{1}$$

$$D_j^{(k)} = e_{15}^{(k)} w_j'^{(k)} - \varepsilon_{11}^{(k)} \phi_j'^{(k)} \tag{2}$$

where $k = 1, 2, 3, 4$, σ_{jz}, D_j ($j = x, y$) are the anti-plane shear stress and in-plane electric displacement, respectively. $c_{44}^{(k)}$, $e_{15}^{(k)}$, $\varepsilon_{11}^{(k)}$ are the shear modulus, piezoelectric coefficient and dielectric parameter, respectively. $w^{(k)}$ and $\phi^{(k)}$ are the mechanical displacement and electric potential, respectively. where $c_{44}^{(1)} = c_{44}^{(4)}$, $c_{44}^{(2)} = c_{44}^{(3)}$, $e_{15}^{(1)} = e_{15}^{(4)}$, $e_{15}^{(2)} = e_{15}^{(3)}$, $\varepsilon_{11}^{(1)} = \varepsilon_{11}^{(4)}$, $\varepsilon_{11}^{(2)} = \varepsilon_{11}^{(3)}$.

The anti-plane governing equations are

$$c_{44}^{(k)} \nabla^2 w^{(k)} + e_{15}^{(k)} \nabla^2 \phi^{(k)} = 0 \tag{3}$$

$$e_{15}^{(k)} \nabla^2 w^{(k)} - \varepsilon_{11}^{(k)} \nabla^2 \phi^{(k)} = 0 \tag{4}$$

where $k = 1, 2, 3, 4$, $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$ is the two dimensional Laplace operator. Since no opening displacement exists for the present anti-plane problem, the crack surfaces can be assumed to be in perfect contact. Accordingly, permeable condition will be enforced in the present study, i.e., both the electric potential and the normal electric displacement are assumed to be continuous across the crack surfaces. The problem demonstrated in Fig. 1 will be solved under the following boundary conditions (In this paper, we just consider the perturbation stress and the perturbation electric displacement field.)

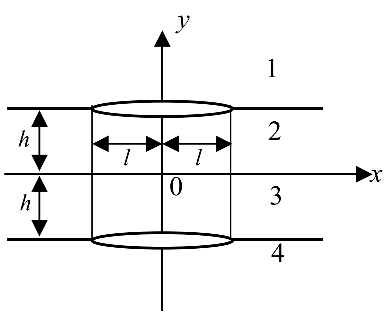

Fig. 1. Parallel interface cracks in the piezoelectric materials.

$$\sigma_{yz}^{(1)}(x,h^+) = \sigma_{yz}^{(2)}(x,h^-), \quad D_y^{(1)}(x,h^+) = D_y^{(2)}(x,h^-), \qquad |x| > l \tag{5}$$

$$w^{(1)}(x,h^+) = w^{(2)}(x,h^-), \quad \phi^{(1)}(x,h^+) = \phi^{(2)}(x,h^-), \qquad |x| > l \tag{6}$$

$$\sigma_{yz}^{(1)}(x,h^+) = \sigma_{yz}^{(2)}(x,h^-) = -\tau_0, \quad |x| \leqslant l \tag{7}$$

$$D_y^{(1)}(x,h^+) = D_y^{(2)}(x,h^-), \quad \phi^{(1)}(x,h^+) = \phi^{(2)}(x,h^-), \qquad |x| \leqslant l \tag{8}$$

$$w^{(2)}(x,0^+) = w^{(3)}(x,0^-), \quad \phi^{(2)}(x,0^+) = \phi^{(3)}(x,0^-), \qquad |x| \geqslant 0 \tag{9}$$

$$\sigma_{yz}^{(2)}(x,0^+) = \sigma_{yz}^{(3)}(x,0^-), \quad D_y^{(2)}(x,0^+) = D_y^{(3)}(x,0^-), \qquad |x| \geqslant 0 \tag{10}$$

$$\sigma_{yz}^{(3)}(x,-h^+) = \sigma_{yz}^{(4)}(x,-h^-), \quad D_y^{(3)}(x,-h^+) = D_y^{(4)}(x,-h^-), \qquad |x| > l \tag{11}$$

$$w^{(3)}(x,-h^+) = w^{(4)}(x,-h^-), \quad \phi^{(3)}(x,-h^+) = \phi^{(4)}(x,-h^-), \qquad |x| > l \tag{12}$$

$$\sigma_{yz}^{(3)}(x,-h^+) = \sigma_{yz}^{(4)}(x,-h^-) = -\tau_0, \qquad |x| \leqslant l \tag{13}$$

$$D_y^{(3)}(x,-h^+) = D_y^{(4)}(x,-h^-), \quad \phi^{(3)}(x,-h^+) = \phi^{(4)}(x,h^-), \qquad |x| \leqslant l \tag{14}$$

where τ_0 is the uniform applied shear traction.

3. Solution

The solutions of Eqs. (3) and (4) can be written as

$$\begin{cases} w^{(1)}(x,y) = \frac{2}{\pi}\int_0^\infty A_1(s)\mathrm{e}^{-sy}\cos(sx)\,\mathrm{d}s, \\ \phi^{(1)}(x,y) = \frac{e_{15}^{(1)}}{\varepsilon_{11}^{(1)}} w^{(1)}(x,y) + \frac{2}{\pi}\int_0^\infty B_1(s)\mathrm{e}^{-sy}\cos(sx)\,\mathrm{d}s, \end{cases} \quad y \geqslant h \tag{15}$$

$$\begin{cases} w^{(2)}(x,y) = \frac{2}{\pi}\int_0^\infty [A_2(s)\mathrm{e}^{-sy} + B_2(s)\mathrm{e}^{sy}]\cos(sx)\,\mathrm{d}s, \\ \phi^{(2)}(x,y) = \frac{e_{15}^{(2)}}{\varepsilon_{11}^{(2)}} w^{(2)}(x,y) + \frac{2}{\pi}\int_0^\infty [C_2(s)\mathrm{e}^{-sy} + D_2(s)\mathrm{e}^{sy}]\cos(sx)\,\mathrm{d}s, \end{cases} \quad h \geqslant y \geqslant 0 \tag{16}$$

$$\begin{cases} w^{(3)}(x,y) = \frac{2}{\pi}\int_0^\infty [A_3(s)\mathrm{e}^{sy} + B_3(s)\mathrm{e}^{-sy}]\cos(sx)\,\mathrm{d}s, \\ \phi^{(3)}(x,y) = \frac{e_{15}^{(2)}}{\varepsilon_{11}^{(2)}} w^{(3)}(x,y) + \frac{2}{\pi}\int_0^\infty [C_3(s)\mathrm{e}^{sy} + D_3(s)\mathrm{e}^{-sy}]\cos(sx)\,\mathrm{d}s, \end{cases} \quad 0 \geqslant y \geqslant -h \tag{17}$$

$$\begin{cases} w^{(4)}(x,y) = \frac{2}{\pi}\int_0^\infty A_4(s)\mathrm{e}^{sy}\cos(sx)\,\mathrm{d}s, \\ \phi^{(4)}(x,y) = \frac{e_{15}^{(1)}}{\varepsilon_{11}^{(1)}} w^{(4)}(x,y) + \frac{2}{\pi}\int_0^\infty B_4(s)\mathrm{e}^{sy}\cos(sx)\,\mathrm{d}s, \end{cases} \quad y \leqslant -h \tag{18}$$

where $A_k(s)$, $B_k(s)$ $(k = 1, 2, 3, 4)$, $C_j(s)$ and $D_j(s)$ $(j = 2, 3)$ are unknown functions.

Then, substituting Eqs. (15)–(18) into the equation of equilibrium and charge equation of electrostatics, we obtain

$$\begin{cases} \sigma_{yz}^{(1)}(x,y) = -\mu^{(1)}\frac{2}{\pi}\int_0^\infty sA_1(s)\mathrm{e}^{-sy}\cos(sx)\,\mathrm{d}s - e_{15}^{(1)}\frac{2}{\pi}\int_0^\infty sB_1(s)\mathrm{e}^{-sy}\cos(sx)\,\mathrm{d}s, \\ D_y^{(1)}(x,y) = \varepsilon_{11}^{(1)}\frac{2}{\pi}\int_0^\infty sB_1(s)\mathrm{e}^{-sy}\cos(sx)\,\mathrm{d}s, \end{cases} \quad y \geqslant h \tag{19}$$

$$\begin{aligned} \sigma_{yz}^{(2)}(x,y) = &-\mu^{(2)}\frac{2}{\pi}\int_0^\infty s[A_2(s)\mathrm{e}^{-sy} - B_2(s)\mathrm{e}^{sy}]\cos(sx)\,\mathrm{d}s \\ &- e_{15}^{(2)}\frac{2}{\pi}\int_0^\infty s[C_2(s)\mathrm{e}^{-sy} - D_2(s)\mathrm{e}^{sy}]\cos(sx)\,\mathrm{d}s, \quad h \geqslant y \geqslant 0 \end{aligned} \tag{20}$$

$$D_y^{(2)} = \varepsilon_{11}^{(2)}\frac{2}{\pi}\int_0^\infty s[C_2(s)\mathrm{e}^{-sy} - D_2(s)\mathrm{e}^{sy}]\cos(sx)\,\mathrm{d}s, \quad h \geqslant y \geqslant 0 \tag{21}$$

$$\begin{aligned} \sigma_{yz}^{(3)}(x,y) = &\mu^{(2)}\frac{2}{\pi}\int_0^\infty s[A_3(s)\mathrm{e}^{sy} - B_3(s)\mathrm{e}^{-sy}]\cos(sx)\,\mathrm{d}s \\ &+ e_{15}^{(2)}\frac{2}{\pi}\int_0^\infty s[C_3(s)\mathrm{e}^{sy} - D_3(s)\mathrm{e}^{-sy}]\cos(sx)\,\mathrm{d}s, \quad 0 \geqslant y \geqslant -h \end{aligned} \tag{22}$$

$$D_y^{(3)} = -\varepsilon_{11}^{(2)}\frac{2}{\pi}\int_0^\infty s[C_3(s)\mathrm{e}^{sy} - D_3(s)\mathrm{e}^{-sy}]\cos(sx)\,\mathrm{d}s, \quad 0 \geqslant y \geqslant -h \tag{23}$$

$$\begin{cases} \sigma_{yz}^{(4)}(x,y) = \mu^{(1)}\frac{2}{\pi}\int_0^\infty sA_4(s)\mathrm{e}^{sy}\cos(sx)\,\mathrm{d}s + e_{15}^{(1)}\frac{2}{\pi}\int_0^\infty sB_4(s)\mathrm{e}^{sy}\cos(sx)\,\mathrm{d}s, \\ D_y^{(4)}(x,y) = -\varepsilon_{11}^{(1)}\frac{2}{\pi}\int_0^\infty sB_4(s)\mathrm{e}^{sy}\cos(sx)\,\mathrm{d}s, \end{cases} \quad y \leqslant -h \tag{24}$$

where $\mu^{(k)} = c_{44}^{(k)} + e_{15}^{(k)^2}/\varepsilon_{11}^{(k)}$.

Using boundary conditions (5)–(14), we obtain

$$-\mu^{(1)}A_1(s)\mathrm{e}^{-sh} - e_{15}^{(1)}B_1(s)\mathrm{e}^{-sh} = -\mu^{(2)}[A_2(s)\mathrm{e}^{-sh} - B_2(s)\mathrm{e}^{sh}] - e_{15}^{(2)}[C_2(s)\mathrm{e}^{-sh} - D_2(s)\mathrm{e}^{sh}] \tag{25}$$

$$\mu^{(2)}[A_3(s)\mathrm{e}^{-sh} - B_3(s)\mathrm{e}^{sh}] + e_{15}^{(2)}[C_3(s)\mathrm{e}^{-sh} - D_3(s)\mathrm{e}^{sh}] = \mu^{(1)}A_4(s)\mathrm{e}^{-sh} + e_{15}^{(1)}B_4(s)\mathrm{e}^{-sh} \tag{26}$$

$$\varepsilon_{11}^{(1)}B_1(s)\mathrm{e}^{-sh} = \varepsilon_{11}^{(2)}[C_2(s)\mathrm{e}^{-sh} - D_2(s)\mathrm{e}^{sh}] \tag{27}$$

$$-\varepsilon_{11}^{(2)}[C_3(s)\mathrm{e}^{-sh} - D_3(s)\mathrm{e}^{sh}] = -\varepsilon_{11}^{(1)}B_4(s)\mathrm{e}^{-sh} \tag{28}$$

$$A_2(s) + B_2(s) = A_3(s) + B_3(s), \quad C_2(s) + D_2(s) = C_3(s) + D_3(s) \tag{29a}$$

$$A_2(s) - B_2(s) = -A_3(s) + B_3(s), \quad C_2(s) - D_2(s) = -C_3(s) + D_3(s) \tag{29b}$$

The gap functions of the crack surface displacements and the electric potentials are defined as follows:

$$f_1(x) = w^{(1)}(x,h^+) - w^{(2)}(x,h^-) \tag{30}$$

$$f_{\phi 1}(x) = \phi^{(1)}(x,h^+) - \phi^{(2)}(x,h^-) \tag{31}$$

$$f_2(x) = w^{(3)}(x,-h^+) - w^{(4)}(x,-h^-) \tag{32}$$

$$f_{\phi 2}(x) = \phi^{(3)}(x,-h^+) - \phi^{(4)}(x,-h^-) \tag{33}$$

Substituting Eqs. (15)–(18) into Eqs. (30)–(33), and applying the Fourier transform, it can be obtained

$$\bar{f}_1(s) = A_1(s)e^{-sh} - A_2(s)e^{-sh} - B_2(s)e^{sh} \tag{34}$$

$$\bar{f}_{\phi 1}(s) = \frac{e_{15}^{(1)}}{\varepsilon_{11}^{(1)}}A_1(s)e^{-sh} - \frac{e_{15}^{(2)}}{\varepsilon_{11}^{(2)}}A_2(s)e^{-sh} - \frac{e_{15}^{(2)}}{\varepsilon_{11}^{(2)}}B_2(s)e^{sh} + B_1(s)e^{-sh} - C_2(s)e^{-sh} - D_2(s)e^{sh} = 0 \tag{35}$$

$$\bar{f}_2(s) = A_3(s)e^{-sh} + B_3(s)e^{sh} - A_4(s)e^{-sh} \tag{36}$$

$$\bar{f}_{\phi 2}(s) = \frac{e_{15}^{(2)}}{\varepsilon_{11}^{(2)}}A_3(s)e^{-sh} + \frac{e_{15}^{(2)}}{\varepsilon_{11}^{(2)}}B_3(s)e^{sh} + C_3(s)e^{-sh} + D_3(s)e^{sh} - \frac{e_{15}^{(1)}}{\varepsilon_{11}^{(1)}}A_4(s)e^{-sh} - B_4(s)e^{-sh} = 0 \tag{37}$$

A superposed bar indicates the Fourier transform throughout the paper. By solving 12 Eqs. (25)–(28), (29a), (29b) and (34)–(37) with 12 unknown functions and substituting the solutions into Eqs. (19) and (24) and applying the boundary conditions (7) and (13), it can be obtained (see Appendix A):

$$\frac{2}{\pi}\int_0^\infty \bar{f}_1(s)\cos(sx)\,ds = 0, \quad |x| > l \tag{38}$$

$$\frac{2}{\pi}\int_0^\infty \bar{f}_2(s)\cos(sx)\,ds = 0, \quad |x| > l \tag{39}$$

$$\frac{2}{\pi}\int_0^\infty s[FG(s)\bar{f}_1(s) + FI(s)\bar{f}_2(s)]\cos(sx)\,ds = -\tau_0, \quad |x| \leqslant l \tag{40}$$

$$\frac{2}{\pi}\int_0^\infty s[FI(s)\bar{f}_1(s) + FG(s)\bar{f}_2(s)]\cos(sx)\,ds = -\tau_0, \quad |x| \leqslant l \tag{41}$$

and

$$\bar{f}_{\phi 1}(s) = 0, \quad \bar{f}_{\phi 2}(s) = 0, \quad f_{\phi 1}(x) = 0, \quad f_{\phi 2}(x) = 0 \qquad \text{for all } s \text{ and } x \tag{42}$$

where $FG(s)$, $\lim_{s\to\infty} FG(s) = FGC$ and $FI(s)$ are known functions. FGC is a constant (see Appendix A).

From Eqs. (38)–(41), it can be obtained

$$\bar{f}_1(s) = \bar{f}_2(s) \Rightarrow f_1(x) = f_2(x), \quad \sigma_{yz}^{(1)}(x,h) = \sigma_{yz}^{(2)}(x,h) = \sigma_{yz}^{(3)}(x,-h) = \sigma_{yz}^{(4)}(x,-h) \tag{43}$$

$$D_y^{(1)}(x,h) = D_y^{(2)}(x,h) = D_y^{(3)}(x,-h) = D_y^{(4)}(x,-h) \tag{44}$$

To determine the unknown functions $\bar{f}_1(s)$ and $\bar{f}_2(s)$, the above two pairs of dual integral equations (38)–(41) must be solved.

4. Solution of the dual integral equation

The Schmidt method (Morse and Feshbach, 1958) is used to solve the dual integral equations. The gap function of the crack surface displacement is represented by the following series:

$$f_1(x) = f_2(x) = \sum_{n=1}^{\infty} a_n P_{2n-2}^{(1/2,1/2)}\left(\frac{x}{l}\right)\left(1 - \frac{x^2}{l^2}\right)^{1/2}, \quad \text{for } -l \leqslant x \leqslant l, \ y = 0 \tag{45}$$

$$f_1(x) = f_2(x) = w^{(1)}(x,h^+) - w^{(2)}(x,h^-) = 0, \quad \text{for } |x| > l, \ y = 0 \tag{46}$$

where a_n are unknown coefficients to be determined and $P_n^{(1/2,1/2)}(x)$ is a Jacobi polynomial (Gradshteyn and Ryzhik, 1980). The Fourier transform of (45) and (46) is (Erdelyi, 1954)

$$\bar{f}_1(s) = \sum_{n=1}^{\infty} a_n G_n \frac{1}{s} J_{2n-1}(sl), \quad G_n = 2\sqrt{\pi}(-1)^{n-1} \frac{\Gamma\left(2n - \frac{1}{2}\right)}{(2n-2)!} \tag{47}$$

where $\Gamma(x)$ and $J_n(x)$ are the Gamma and Bessel functions, respectively.

Substituting (47) into Eqs. (38)–(41), respectively. It can be shown that Eqs. (38) and (39) are automatically satisfied. After integration with respect to x in $[-l, x]$, Eq. (40) reduces to

$$\frac{2}{\pi} \sum_{n=1}^{\infty} a_n G_n \int_0^{\infty} \frac{1}{s} [FG(s) + FI(s)] J_{2n-1}(sl) \sin(sx)\, \mathrm{d}s = -\tau_0 x \tag{48}$$

The semi-infinite integral in (35) can be modified as (Gradshteyn and Ryzhik, 1980)

$$\int_0^{\infty} \frac{1}{s} J_n(sa) \sin(bs)\, \mathrm{d}s = \begin{cases} \dfrac{\sin[n \sin^{-1}(b/a)]}{n}, & a > b \\ \dfrac{a^n \sin(n\pi/2)}{n[b + \sqrt{b^2 - a^2}]^n}, & b > a \end{cases} \tag{49}$$

The semi-infinite integral in Eq. (48) can be modified as:

$$\int_0^{\infty} \frac{1}{s} [FG(s) + FI(s)] J_{2n-1}(sl) \sin(sx)\, \mathrm{d}s = \frac{FGC}{2n-1} \sin\left[(2n-1)\sin^{-1}\left(\frac{x}{l}\right)\right] + \int_0^{\infty} \frac{1}{s} [FG(s) - FGC + FI(s)] J_{2n-1}(sl) \sin(sx)\, \mathrm{d}s \tag{50}$$

Thus the semi-infinite integral in (48) can be evaluated directly. Eq. (48) can now be solved for the coefficients a_n by the Schmidt method (Morse and Feshbach, 1958). For brevity, (48) can be rewritten as

$$\sum_{n=1}^{\infty} a_n E_n(x) = U(x), \quad -l < x < l \tag{51}$$

where $E_n(x)$ and $U(x)$ are known functions and the coefficients a_n are to be determined. A set of functions $P_n(x)$ which satisfy the orthogonality condition

$$\int_{-l}^{l} P_m(x) P_n(x)\, \mathrm{d}x = N_n \delta_{mn}, \quad N_n = \int_{-l}^{l} P_n^2(x)\, \mathrm{d}x \tag{52}$$

can be constructed from the function, $E_n(x)$, such that

$$P_n(x) = \sum_{i=1}^{n} \frac{M_{in}}{M_{nn}} E_i(x) \tag{53}$$

where M_{ij} is the co-factor of the element d_{ij} of D_n, which is defined as

$$D_n = \begin{bmatrix} d_{11}, d_{12}, d_{13}, \ldots, d_{1n} \\ d_{21}, d_{22}, d_{23}, \ldots, d_{2n} \\ d_{31}, d_{32}, d_{33}, \ldots, d_{3n} \\ \cdots \\ \cdots \\ \cdots \\ d_{n1}, d_{n2}, d_{n3}, \ldots, d_{nn} \end{bmatrix}, \quad d_{ij} = \int_{-l}^{l} E_i(x) E_j(x)\, \mathrm{d}x \tag{54}$$

Using Eqs. (51)–(54), we obtain

$$a_n = \sum_{j=n}^{\infty} q_j \frac{M_{nj}}{M_{jj}} \quad \text{with } q_j = \frac{1}{N_j} \int_{-l}^{l} U(x) P_j(x)\, \mathrm{d}x \tag{55}$$

5. Intensity factors

The coefficients a_n are known, so that the entire perturbation stress field and the perturbation electric displacement can be obtained. However, in fracture mechanics, it is of importance to determine the perturbation stress σ_{yz} and the perturbation electric displacement D_y in the vicinity of the crack's tips. $\sigma_{yz}^{(1)}$, $\sigma_{yz}^{(2)}$, $\sigma_{yz}^{(3)}$, $\sigma_{yz}^{(4)}$, $D_y^{(1)}$, $D_y^{(2)}$, $D_y^{(3)}$ and $D_y^{(4)}$ along the crack line can be expressed respectively as

$$\begin{aligned}\sigma_{yz}^{(1)}(x,h) &= \sigma_{yz}^{(2)}(x,h) = \sigma_{yz}^{(3)}(x,-h) = \sigma_{yz}^{(4)}(x,-h) = \sigma_{yz} \\ &= \frac{2}{\pi} \sum_{n=1}^{\infty} a_n G_n \int_0^{\infty} \{FGC + [FG(s) - FGC + FI(s)]\} J_{2n-1}(sl) \cos(xs)\, \mathrm{d}s\end{aligned} \tag{56}$$

$$\begin{aligned}D_y^{(1)}(x,h) &= D_y^{(2)}(x,h) = D_y^{(3)}(x,-h) = D_y^{(4)}(x,-h) = D_y \\ &= \frac{2}{\pi} \sum_{n=1}^{\infty} a_n G_n \int_0^{\infty} \{FLC + [FL(s) - FLC]\} J_{2n-1}(sl) \cos(xs)\, \mathrm{d}s\end{aligned} \tag{57}$$

where $FL(s)$ is a known function. FLC is a constant (see Appendix A).

An examination of Eqs. (56) and (57), the singular part of the stress field and the singular part of the electric displacement can be obtained respectively from the relationship (Gradshteyn and Ryzhik, 1980):

$$\int_0^{\infty} J_n(sa) \cos(bs)\, \mathrm{d}s = \begin{cases} \dfrac{\cos[n \sin^{-1}(b/a)]}{\sqrt{a^2 - b^2}}, & a > b \\ -\dfrac{a^n \sin(n\pi/2)}{\sqrt{b^2 - a^2}[b + \sqrt{b^2 - a^2}]^n}, & b > a \end{cases} \tag{58}$$

The singular part of the stress field and the singular part of the electric displacement can be expressed respectively as follows ($l < x$):

$$\tau = -\frac{2FGC}{\pi} \sum_{n=1}^{\infty} a_n G_n H_n(x) \tag{59}$$

$$D = -\frac{2FLC}{\pi} \sum_{n=1}^{\infty} a_n G_n H_n(x) \tag{60}$$

where

$$H_n(x) = \frac{(-1)^{n-1} l^{2n-1}}{\sqrt{x^2 - l^2}[x + \sqrt{x^2 - l^2}]^{2n-1}}$$

We obtain the stress intensity factor K as

$$K = \lim_{x \to l^+} \sqrt{2\pi(x - l)} \cdot \tau = -\frac{4FGC}{\sqrt{l}} \sum_{n=0}^{\infty} a_n \frac{\Gamma\left(2n - \frac{1}{2}\right)}{(2n-2)!} \tag{61}$$

We obtain the electric displacement intensity factor K^D as

$$K^D = \lim_{x \to l^+} \sqrt{2\pi(x-l)} \cdot D = -\frac{4FLC}{\sqrt{l}} \sum_{n=0}^{\infty} a_n \frac{\Gamma\left(2n-\frac{1}{2}\right)}{(2n-2)!} = \frac{FLC}{FGC} K \tag{62}$$

6. Numerical calculations and discussion

This section presents numerical results of several representative problems. Adopting the first 10 terms in the infinite series (51), we followed the Schmidt procedure. From the literatures (see e.g. Itou, 1978; Zhou et al., 1999a,b), it can be seen that the Schmidt method performs satisfactorily if the first 10 terms of the infinite series (51) are retained. The solution does not change with an increase of the number of terms in (51) beyond 10. The precision of present solution can satisfy the demands of the practical problem. The piezoelectric layer and the piezoelectric half plane are assumed to be the commercially available piezoelectric PZT-4 or PZT-5H, respectively. The engineering material constants of PZT-4 are $c_{44} = 2.56(\times 10^{10}\ \text{N/m}^2)$, $e_{15} = 12.7\ (\text{c/m}^2)$, $\varepsilon_{11} = 64.6(\times 10^{-10}\ \text{c/V m}^2)$, respectively. The material constants of PZT-5H are $c_{44} = 2.3(\times 10^{10}\ \text{N/m}^2)$, $e_{15} = 17.0\ (\text{c/m}^2)$, $\varepsilon_{11} = 150.4(\times 10^{-10}\ \text{c/V m}^2)$, respectively. The results of the present paper are shown in Figs. 2–9. From the results, the following observations are very significant:

(i) The stress and the electric displacement intensity factors not only depend on the crack length and the width of the piezoelectric layer, but also on the properties of the materials.

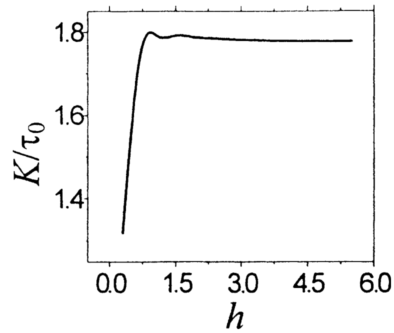

Fig. 2. The stress intensity factor versus h for $l = 1.0$ (materials of the upper and the lower half planes are PZT-4 and the material of the interlayer is PZT-5H).

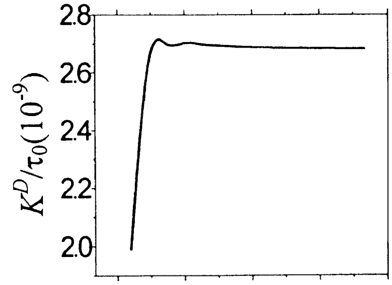

Fig. 3. The electric displacement intensity factor versus h for $l = 1.0$ (materials of the upper and the lower half planes are PZT-4 and the material of the interlayer is PZT-5H).

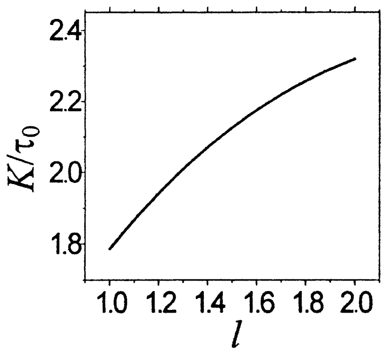

Fig. 4. The stress intensity factor versus l for $h = 1.0$ (materials of the upper and the lower half planes are PZT-4 and the material of the interlayer is PZT-5H).

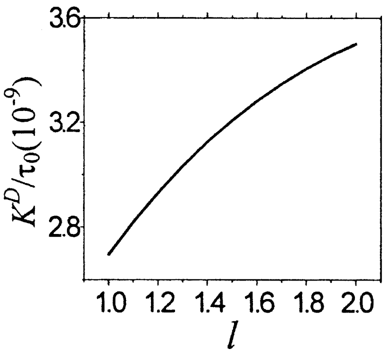

Fig. 5. The electric displacement intensity factor versus l for $h = 1.0$ (materials of the upper and the lower half planes are PZT-4 and the material of the interlayer is PZT-5H).

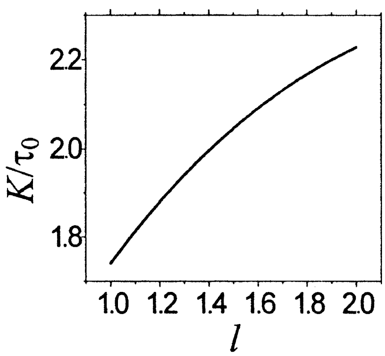

Fig. 6. The stress intensity factor versus l for $h = 1.0$ (materials of the upper and the lower half planes are PZT-5H and the material of the interlayer is PZT-4).

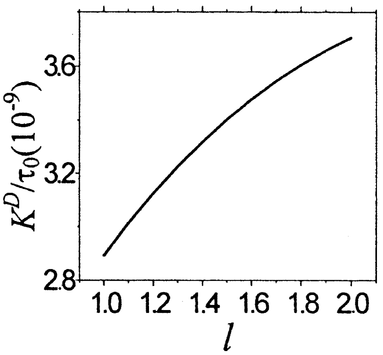

Fig. 7. The electric displacement intensity factor versus l for $h = 1.0$ (materials of the upper and the lower half planes are PZT-5H and the material of the interlayer is PZT-4).

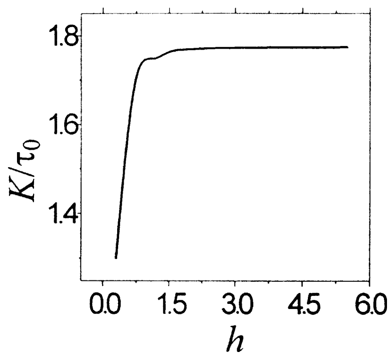

Fig. 8. The stress intensity factor versus h for $l = 1.0$ (materials of the upper and the lower half planes are PZT-5H and the material of the interlayer is PZT-4).

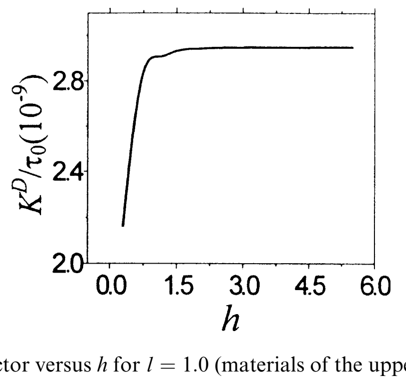

Fig. 9. The electric displacement intensity factor versus h for $l = 1.0$ (materials of the upper and the lower half planes are PZT-5H and the material of the interlayer is PZT-4).

(ii) In contrast to the impermeable crack surface condition solution, it is found that the perturbation electric displacement intensity factor for the permeable crack surface conditions is much smaller than the results for the impermeable crack surface conditions as shown in Zhou's paper (Zhou and Shen, 1999).

(iii) The stress and the electric displacement intensity factors increases as the distance between the parallel cracks increases as shown in Figs. 2, 3, 8, and 9. This phenomenon is called crack shielding effect as discussed in Ratwani's paper (Ratwani and Gupta, 1974). The stress intensity factors increases as the distance between the parallel cracks increases. However, the shield effects will be very small for $h > 2.0$.

(iv) The stress intensity factors and the electric displacement intensity factors of the parallel cracks increase as the length of cracks increases as shown in Figs. 4–7. Noting this fact, experiments indicate that the piezoelectric materials with smaller cracks are more resistant to fracture than those with larger cracks.

(v) On the other hand, one should bear in mind that the large stress and electric field concentration in the region around the crack tips may induce domain switching effects as discussed in Fulton's paper (Fulton and Gao, 1997). Such nonlinear effect may change the results greatly. In the present analysis, we neglect such effect, and only consider the linear behavior of the materials.

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Appendix A

$$H_1 = (1 - e^{-4hs}), \quad H_2 = (1 + e^{-4hs}), \quad F_1 = H_1^2 \varepsilon_{11}^{(1)^4} e_{15}^{(2)^4}, \quad F_2 = -4H_1^2 \varepsilon_{11}^{(1)^3} \varepsilon_{11}^{(2)} e_{15}^{(1)} e_{15}^{(2)^3},$$

$$F_3 = -2H_1 \varepsilon_{11}^{(1)^4} \varepsilon_{11}^{(2)} e_{15}^{(2)^2} [H_2\mu_1 + H_1\mu_2], \quad F_4 = H_1^2 \varepsilon_{11}^{(2)^4} e_{15}^{(1)^4},$$

$$F_5 = -2H_1 \varepsilon_{11}^{(1)} \varepsilon_{11}^{(2)^4} e_{15}^{(1)^2} [H_1\mu_1 + H_2\mu_2], \quad F_6 = H_1 \varepsilon_{11}^{(1)^2} \varepsilon_{11}^{(2)^4} [H_1\mu_1^2 + 2H_2\mu_1\mu_2 + H_1\mu_2^2],$$

$$F_7 = -4H_1^2 \varepsilon_{11}^{(1)} \varepsilon_{11}^{(2)^3} e_{15}^{(1)^3} e_{15}^{(2)}, \quad F_8 = -2H_1 \varepsilon_{11}^{(1)^2} \varepsilon_{11}^{(2)^3} e_{15}^{(1)^2} [H_2\mu_1 + H_1\mu_2],$$

$$F_9 = 4H_1 \varepsilon_{11}^{(1)^2} \varepsilon_{11}^{(2)^3} e_{15}^{(1)} e_{15}^{(2)} [H_1\mu_1 + H_2\mu_2], \quad F_{10} = 2H_1 \varepsilon_{11}^{(1)^3} \varepsilon_{11}^{(2)^3} [H_1\mu_1^2 + 2H_2\mu_1\mu_2 + H_1\mu_2^2],$$

$$F_{11} = 6H_1^2 \varepsilon_{11}^{(1)^2} \varepsilon_{11}^{(2)^2} e_{15}^{(1)^2} e_{15}^{(2)^2}, \quad F_{12} = 4H_1 \varepsilon_{11}^{(1)^3} \varepsilon_{11}^{(2)^2} e_{15}^{(1)} e_{15}^{(2)} [H_2\mu_1 + H_1\mu_2],$$

$$F_{13} = -2H_1 \varepsilon_{11}^{(1)^3} \varepsilon_{11}^{(2)^2} e_{15}^{(2)^2} [H_1\mu_1 + H_2\mu_2], \quad F_{14} = H_1 \varepsilon_{11}^{(1)^4} \varepsilon_{11}^{(2)^2} [H_1\mu_1^2 + 2H_2\mu_1\mu_2 + H_1\mu_2^2],$$

$$FF = F_1 + F_2 + F_3 + F_4 + F_5 + F_6 + F_7 + F_8 + F_9 + F_{10} + F_{11} + F_{12} + F_{13} + F_{14},$$

$$G_1 = -H_1 \varepsilon_{11}^{(2)^4} e_{15}^{(1)^4} e^{-4hs} \mu_2, \quad G_2 = -H_1 \varepsilon_{11}^{(2)^4} e_{15}^{(1)^4} \mu_2,$$

$$G_3 = -H_1^2 \varepsilon_{11}^{(1)^4} \mu_1 (e_{15}^{(2)^2} - \varepsilon_{11}^{(2)} \mu_2) e_{15}^{(2)^2},$$

$$G_4 = H_1 \varepsilon_{11}^{(1)^4} \mu_1 (e_{15}^{(2)^2} - \varepsilon_{11}^{(2)} \mu_2) \varepsilon_{11}^{(2)} (\mu_1 H_2 + \mu_2 H_1), \quad G_5 = 2H_1 H_2 \varepsilon_{11}^{(1)} \varepsilon_{11}^{(2)^3} \mu_2 e_{15}^{(1)^3} e_{15}^{(2)},$$

$$G_6 = H_1 \varepsilon_{11}^{(1)} \varepsilon_{11}^{(2)^4} \mu_2 e_{15}^{(1)^2} (2H_2\mu_1 + H_1\mu_2), \quad G_7 = 2H_1^2 \varepsilon_{11}^{(1)^3} \varepsilon_{11}^{(2)} \mu_1 (e_{15}^{(2)^2} - \varepsilon_{11}^{(2)} \mu_2) e_{15}^{(1)} e_{15}^{(2)},$$

$$G_8 = H_1 \varepsilon_{11}^{(1)^3} \varepsilon_{11}^{(2)^2} \mu_1 e_{15}^{(2)^2} (2H_2\mu_2 + H_1\mu_1), \quad G_9 = -2H_2 \varepsilon_{11}^{(1)^3} \varepsilon_{11}^{(2)^3} \mu_1\mu_2(\mu_1 H_2 + \mu_2 H_1),$$

$$G_{10} = -2H_1H_2 \varepsilon_{11}^{(1)^2} \varepsilon_{11}^{(2)^3} \mu_1\mu_2 e_{15}^{(1)} e_{15}^{(2)}, \quad G_{11} = -H_1 \varepsilon_{11}^{(1)^2} \varepsilon_{11}^{(2)^4} \mu_1\mu_2(H_2\mu_1 + H_1\mu_2),$$

$$G_{12} = H_2 \varepsilon_{11}^{(1)^2} \varepsilon_{11}^{(2)^3} e_{15}^{(1)^2} \mu_2(2H_2\mu_1 + H_1\mu_2), \quad G_{13} = -H_1 \varepsilon_{11}^{(1)^2} \varepsilon_{11}^{(2)^2} e_{15}^{(1)^2} e_{15}^{(2)^2} (\mu_1 H_1 + \mu_2 H_2),$$

$$GG = G_1 + G_2 + G_3 + G_4 + G_5 + G_6 + G_7 + G_8 + G_9 + G_{10} + G_{11} + G_{12} + G_{13},$$

$$FG(s) = GG/FF,$$

$$I_1 = -2H_1 \varepsilon_{11}^{(2)^4} e_{15}^{(1)^4} \mathrm{e}^{-2hs}\mu_2, \quad I_2 = 2H_1 \varepsilon_{11}^{(1)^4} \varepsilon_{11}^{(2)} \mu_1^2 (e_{15}^{(2)^2} - \varepsilon_{11}^{(2)}\mu_2)\mathrm{e}^{-2hs},$$

$$I_3 = 4H_1 \varepsilon_{11}^{(1)} \varepsilon_{11}^{(2)^3} e_{15}^{(1)^3} e_{15}^{(2)} \mathrm{e}^{-2hs}\mu_2, \quad I_4 = 4H_1 \varepsilon_{11}^{(1)} \varepsilon_{11}^{(2)^4} e_{15}^{(1)^2} \mathrm{e}^{-2hs}\mu_1\mu_2,$$

$$I_5 = -4H_2 \varepsilon_{11}^{(1)^3} \varepsilon_{11}^{(2)^3} \mathrm{e}^{-2hs}\mu_1^2\mu_2, \quad I_6 = -4H_1 \varepsilon_{11}^{(1)^2} \varepsilon_{11}^{(2)^3} e_{15}^{(1)} e_{15}^{(2)} \mathrm{e}^{-2hs}\mu_1\mu_2,$$

$$I_7 = -2H_1 \varepsilon_{11}^{(1)^2} \varepsilon_{11}^{(2)^4} \mathrm{e}^{-2hs}\mu_1^2\mu_2, \quad I_8 = 2H_1 \varepsilon_{11}^{(1)^2} \varepsilon_{11}^{(2)^3} e_{15}^{(1)^2} \mathrm{e}^{-2hs}\mu_2(2H_2\mu_1 + H_1\mu_2),$$

$$I_9 = -2H_1 \varepsilon_{11}^{(1)^2} \varepsilon_{11}^{(2)^2} e_{15}^{(1)^2} e_{15}^{(2)^2} \mathrm{e}^{-2hs}\mu_2,$$

$$II = I_1 + I_2 + I_3 + I_4 + I_5 + I_6 + I_7 + I_8 + I_9,$$

$$FI(s) = II/FF,$$

$$L_1 = H_1 \varepsilon_{11}^{(1)} \varepsilon_{11}^{(2)^4} e_{15}^{(1)^3} (\mathrm{e}^{-4hs} + 2\mathrm{e}^{-2hs} + 1)\mu_2, \quad L_2 = H_1^2 \varepsilon_{11}^{(1)^4} \varepsilon_{11}^{(2)} e_{15}^{(2)^3} \mu_1,$$

$$L_3 = -2H_1 \varepsilon_{11}^{(1)^4} \varepsilon_{11}^{(2)^2} e_{15}^{(2)} \mathrm{e}^{-2hs}\mu_1^2, \quad L_4 = -H_1 \varepsilon_{11}^{(1)^4} \varepsilon_{11}^{(2)^2} e_{15}^{(2)} \mu_1(\mu_1 H_2 + \mu_2 H_1),$$

$$L_5 = -H_1 \varepsilon_{11}^{(1)^2} \varepsilon_{11}^{(2)^4} e_{15}^{(1)} \mu_2(\mu_1 H_2 + \mu_2 H_1 + 2\mu_1 \mathrm{e}^{-2hs}),$$

$$L_6 = -H_1 \varepsilon_{11}^{(1)^2} \varepsilon_{11}^{(2)^3} e_{15}^{(1)^2} e_{15}^{(2)} (-\mu_1 H_1 + 4\mu_2 \mathrm{e}^{-2hs} + 2\mu_2 H_2),$$

$$L_7 = -H_1 \varepsilon_{11}^{(1)^3} \varepsilon_{11}^{(2)^3} e_{15}^{(2)} \mu_1(\mu_1 H_1 - 2\mu_2 \mathrm{e}^{-2hs} + \mu_2 H_2),$$

$$L_8 = -H_1 \varepsilon_{11}^{(1)^3} \varepsilon_{11}^{(2)^2} e_{15}^{(1)} e_{15}^{(2)^2} (2\mu_1 H_1 - 2\mu_2 \mathrm{e}^{-2hs} - \mu_2 H_2),$$

$$L_9 = -2\varepsilon_{11}^{(1)^3} \varepsilon_{11}^{(2)^3} e_{15}^{(1)} \mu_2 \mathrm{e}^{-2hs}(2\mu_1 H_2 + \mu_2 H_1),$$

$$L_{10} = -\varepsilon_{11}^{(1)^3} \varepsilon_{11}^{(2)^3} e_{15}^{(1)} \mu_2[(1 + 6\mathrm{e}^{-4hs} + \mathrm{e}^{-8hs})\mu_1 + (1 - \mathrm{e}^{-8hs})\mu_2],$$

$$LL = L_1 + L_2 + L_3 + L_4 + L_5 + L_6 + L_7 + L_8 + L_9 + L_{10},$$

$$FL(s) = LL/FF,$$

$$F_{1c} = \varepsilon_{11}^{(1)^4} e_{15}^{(2)^4}, \quad F_{2c} = -4\varepsilon_{11}^{(1)^3} \varepsilon_{11}^{(2)} e_{15}^{(1)} e_{15}^{(2)^3}, \quad F_{3c} = -2\varepsilon_{11}^{(1)^4} \varepsilon_{11}^{(2)} e_{15}^{(2)^2} (\mu_1 + \mu_2), \quad F_{4c} = \varepsilon_{11}^{(2)^4} e_{15}^{(1)^4},$$

$$F_{5c} = -2\varepsilon_{11}^{(1)}\varepsilon_{11}^{(2)^4}e_{15}^{(1)^2}(\mu_1+\mu_2), \quad F_{6c} = \varepsilon_{11}^{(1)^2}\varepsilon_{11}^{(2)^4}(\mu_1^2+2\mu_1\mu_2+\mu_2^2), \quad F_{7c} = -4\varepsilon_{11}^{(1)}\varepsilon_{11}^{(2)^3}e_{15}^{(1)^3}e_{15}^{(2)},$$

$$F_{8c} = -2\varepsilon_{11}^{(1)^2}\varepsilon_{11}^{(2)^3}e_{15}^{(1)^2}(\mu_1+\mu_2), \quad F_{9c} = 4\varepsilon_{11}^{(1)^2}\varepsilon_{11}^{(2)^3}e_{15}^{(1)}e_{15}^{(2)}(\mu_1+\mu_2),$$

$$F_{10c} = 2\varepsilon_{11}^{(1)^3}\varepsilon_{11}^{(2)^3}(\mu_1^2+2\mu_1\mu_2+\mu_2^2), \quad F_{11c} = 6\varepsilon_{11}^{(1)^2}\varepsilon_{11}^{(2)^2}e_{15}^{(1)^2}e_{15}^{(2)^2},$$

$$F_{12c} = 4\varepsilon_{11}^{(1)^3}\varepsilon_{11}^{(2)^2}e_{15}^{(1)}e_{15}^{(2)}(\mu_1+\mu_2),$$

$$F_{13c} = -2\varepsilon_{11}^{(1)^3}\varepsilon_{11}^{(2)^2}e_{15}^{(2)^2}(\mu_1+\mu_2), \quad F_{14c} = \varepsilon_{11}^{(1)^4}\varepsilon_{11}^{(2)^2}(\mu_1^2+2\mu_1\mu_2+\mu_2^2),$$

$$FFC = F_{1c}+F_{2c}+F_{3c}+F_{4c}+F_{5c}+F_{6c}+F_{7c}+F_{8c}+F_{9c}+F_{10c}+F_{11c}+F_{12c}+F_{13c}+F_{14c},$$

$$G_{2c} = -\varepsilon_{11}^{(2)^4}e_{15}^{(1)^4}\mu_2, \quad G_{3c} = -\varepsilon_{11}^{(1)^4}\mu_1(e_{15}^{(2)^2}-\varepsilon_{11}^{(2)}\mu_2)e_{15}^{(2)^2},$$

$$G_{4c} = \varepsilon_{11}^{(1)^4}\mu_1(e_{15}^{(2)^2}-\varepsilon_{11}^{(2)}\mu_2)\varepsilon_{11}^{(2)}(\mu_1+\mu_2), \quad G_{5c} = 2\varepsilon_{11}^{(1)}\varepsilon_{11}^{(2)^3}\mu_2 e_{15}^{(1)^3}e_{15}^{(2)},$$

$$G_{6c} = \varepsilon_{11}^{(1)}\varepsilon_{11}^{(2)^4}\mu_2 e_{15}^{(1)^2}(2\mu_1+\mu_2), \quad G_{7c} = 2\varepsilon_{11}^{(1)^3}\varepsilon_{11}^{(2)}\mu_1(e_{15}^{(2)^2}-\varepsilon_{11}^{(2)}\mu_2)e_{15}^{(1)}e_{15}^{(2)},$$

$$G_{8c} = \varepsilon_{11}^{(1)^3}\varepsilon_{11}^{(2)^2}\mu_1 e_{15}^{(2)^2}(2\mu_2+\mu_1), \quad G_{9c} = -2\varepsilon_{11}^{(1)^3}\varepsilon_{11}^{(2)^3}\mu_1\mu_2(\mu_1+\mu_2),$$

$$G_{10c} = -2\varepsilon_{11}^{(1)^2}\varepsilon_{11}^{(2)^3}\mu_1\mu_2 e_{15}^{(1)}e_{15}^{(2)}, \quad G_{11c} = -\varepsilon_{11}^{(1)^2}\varepsilon_{11}^{(2)^4}\mu_1\mu_2(\mu_1+\mu_2),$$

$$G_{12c} = \varepsilon_{11}^{(1)^2}\varepsilon_{11}^{(2)^3}e_{15}^{(1)^2}\mu_2(2\mu_1+\mu_2), \quad G_{13c} = -\varepsilon_{11}^{(1)^2}\varepsilon_{11}^{(2)^2}e_{15}^{(1)^2}e_{15}^{(2)^2}(\mu_1+\mu_2),$$

$$GGC = G_{2c}+G_{3c}+G_{4c}+G_{5c}+G_{6c}+G_{7c}+G_{8c}+G_{9c}+G_{10c}+G_{11c}+G_{12c}+G_{13c},$$

$$FGC = GGC/FFC,$$

$$L_{1c} = \varepsilon_{11}^{(1)}\varepsilon_{11}^{(2)^4}e_{15}^{(1)^3}\mu_2, \quad L_{2c} = \varepsilon_{11}^{(1)^4}\varepsilon_{11}^{(2)}e_{15}^{(2)^3}\mu_1, \quad L_{4c} = -\varepsilon_{11}^{(1)^4}\varepsilon_{11}^{(2)^2}e_{15}^{(2)}\mu_1(\mu_1+\mu_2),$$

$$L_{5c} = -\varepsilon_{11}^{(1)^2}\varepsilon_{11}^{(2)^4}e_{15}^{(1)}\mu_2(\mu_1+\mu_2), \quad L_{6c} = -\varepsilon_{11}^{(1)^2}\varepsilon_{11}^{(2)^3}e_{15}^{(1)^2}e_{15}^{(2)}(-\mu_1+2\mu_2),$$

$$L_{7c} = -\varepsilon_{11}^{(1)^3}\varepsilon_{11}^{(2)^3}e_{15}^{(2)}\mu_1(\mu_1+\mu_2), \quad L_{8c} = -\varepsilon_{11}^{(1)^3}\varepsilon_{11}^{(2)^2}e_{15}^{(1)}e_{15}^{(2)^2}(2\mu_1-\mu_2),$$

$$L_{10c} = -\varepsilon_{11}^{(1)^3}\varepsilon_{11}^{(2)^3}e_{15}^{(1)}\mu_2(\mu_1+\mu_2),$$

$$LLC = L_{1c}+L_{2c}+L_{4c}+L_{5c}+L_{6c}+L_{7c}+L_{8c}+L_{10c},$$

$$FLC = LLC/FFC,$$

$$\lim_{s\to\infty} FG(s) = FGC, \quad \lim_{s\to\infty} FL(s) = FLC, \quad \lim_{s\to\infty} FI(s) = 0,$$

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